Problem 4A.4

Comparison of exact and approximate profiles for flow along a flat plate. Compare the values of v_x/v_∞ obtained from Eq. 4.4-18 with those from Fig. 4.4-3, at the following values of $y\sqrt{v_\infty/\nu x}$: (a) 1.5, (b) 3.0, (c) 4.0. Express the results as the ratio of the approximate to the exact values.

Answers: (a) 0.96; (b) 0.99; (c) 1.01

Solution

Example 4.4-1 and Example 4.4-2 in the textbook present analyses for the approximate and exact velocity profiles, respectively, for tangental laminar flow along a flat plate.

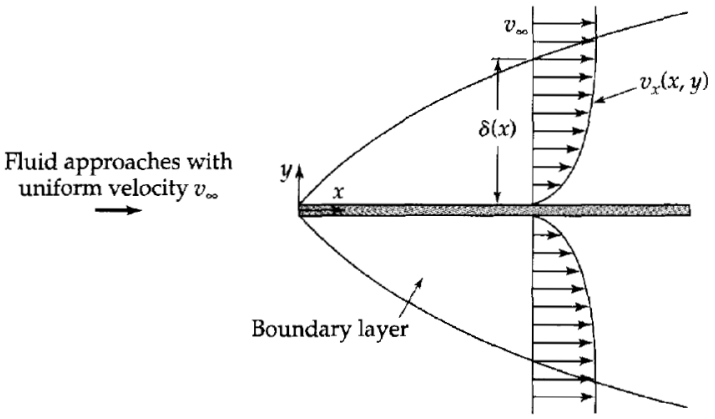

Fig. 4.4-2. Boundary-layer development near a flat plate of negligible thickness.

The approximate formula is given in Eq. 4.4-18,

$$\frac{v_x}{v_\infty} = \frac{3}{2}\left(y\sqrt{\frac{13}{280}\frac{v_\infty}{\nu x}}\right) - \frac{1}{2}\left(y\sqrt{\frac{13}{280}\frac{v_\infty}{\nu x}}\right)^3, \tag{4.4-18}$$

and the exact formula satisfies $v_x/v_\infty = f'$, where

$$f = f(\eta) = f\left(y\sqrt{\frac{v_\infty}{\nu x}}\right)$$

satisfies the Blasius equation,

$$-ff'' = 2f'''$$

subject to the following boundary conditions.

$$\begin{aligned} f(0) &= 0 \\ f'(0) &= 0 \\ \lim_{\eta\to\infty} f'(\eta) &= 1 \end{aligned}$$

A plot of its numerical solution is shown in Fig. 4.4-3.

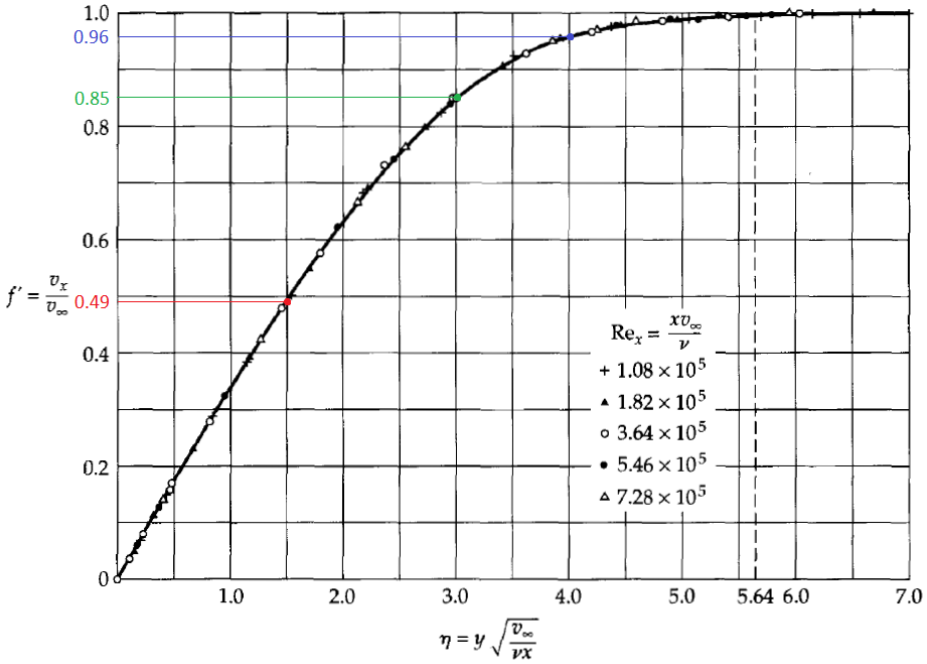

Fig. 4.4-3. Predicted and observed velocity profiles for tangential laminar flow along a flat plate. The solid line represents the solution of Eqs. 4.4-20 to 24, obtained by Blasius [see H. Schlichting, *Boundary-Layer Theory*, McGraw-Hill, New York, 7th edition (1979), p. 137].

Write Eq. 4.4-18 in terms of η

$$\frac{v_x}{v_\infty} = \frac{3}{2}\left(\eta\sqrt{\frac{13}{280}}\right) - \frac{1}{2}\left(\eta\sqrt{\frac{13}{280}}\right)^3$$

and evaluate it at $\eta = 1.5$, $\eta = 3.0$, and $\eta = 4.0$.

$$\eta = 1.5: \quad \frac{v_x}{v_\infty} \approx 0.467932$$

$$\eta = 3.0: \quad \frac{v_x}{v_\infty} \approx 0.834573$$

$$\eta = 4.0: \quad \frac{v_x}{v_\infty} \approx 0.972706$$

Then compare these to the corresponding values from the graph by taking ratios.

$$\eta = 1.5: \quad \frac{0.467932}{0.49} \approx 0.95$$

$$\eta = 3.0: \quad \frac{0.834573}{0.85} \approx 0.98$$

$$\eta = 4.0: \quad \frac{0.972706}{0.96} \approx 1.01$$